Calculus 1 Exam # 1

1. Compute the following limits (20):

   (a)

$$\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 1}$$

   (b)

$$\lim_{x \to 0} \frac{x^3 + 7x^2 - x}{5x^3 + 10x^2 + x}$$

   (c)

$$\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5}$$

(d)

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

2. Solve the following inequality:

$$\frac{1}{x+3} + \frac{1}{x-3} \geq 0.$$

3. Compute

$$\lim_{x \to 0} \frac{\tan 3x}{x^2 + x}$$

4. Evaluate

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

for $f(x) = x^3 + 1$, $c = 1$.

5. Find the domain and range of

$$f(x) = \frac{1}{\sqrt{9 - x^2}}$$

6. Give the definition of limit. Derive the relationship between ϵ and δ that is needed in an $\epsilon - \delta$ proof for the claim

$$\lim_{x \to 1} 2 - 3x = -1$$

7. Use the pinching/sandwich theorem to find

$$\lim_{x \to 0} |x| \cos \frac{1}{x}$$

8. Let f be a function defined on [-100, 100]. We know that if $0 < |x - 3| < 1$ then $|f(x) - 5| < 0.1$. Which of the following statements are true.

   (a) If $|x - 2.5| < 0.3$, then $|f(x) - 5| < 0.1$.

   (b) If $\lim_{x \to 3} f(x) = L$, then $4.9 \leq L \leq 5.1$.

   (c) If $0 < |x - 3| < 1$, then $f(x) - 4.95| < 0.05$.

   (d) If $0 < |x - 3| < 2$, then $f(x) - 5| < 0.1$.

   (e) $f(x)$ is bounded on $[2, 4]$.

9. Classify (removable, jump, infinite) the discontinuities of the following function

$$f(x) = \begin{cases} \frac{1}{x} & \text{for } x > 0 \\ \sqrt{x^2 + 3} & \text{for } -1 < x < 0 \\ x + 3 & \text{for } -2 \leq x < -1 \\ x^2 + 3 & \text{for } x < -2 \end{cases} \tag{1}$$