# Calculus 1 Exam # 3



1. (10) Let $f(x) = \frac{x+5}{x^2-4}$. Find the vertical and horizontal asymptotes of $f$.

2. (10) Find the absolute maximal and minimal values for

$$f(x) = \frac{x}{x^2+4} \qquad x \in [-4, 1].$$

3. (30) Let $f(x) = x^4 - 2x^2$.

   (a) Draw the sign chart of $f'(x)$, determine the local maxima, local minima, increasing intervals and decreasing intervals;

   (b) Find the inflection point and the concave up and concave down intervals ;

   (c) Draw a graph of $f(x)$ by plotting all the local maxima and local minima and inflection points [concavity must be shown on the graph].

4. (15) Given an equilateral triangle with each side of length 2:

   Find the maximal area of the rectangle which can be inscribed in it. Find the dimensions of this rectangle.

5. (15) Find all the inflection points of $f(x) = x^2 + \sin 2x$ for $x \in [0, 2\pi]$. Determine the concave-up, concave-down intervals of $f(x)$ on $[0, 2\pi]$.

6. (10) Determine the local extreme values of

$$F(x) = \sin^2 x - \sqrt{3}\cos x \qquad (0 \leq x \leq \pi).$$

7. (10) Compute the derivative of

$$f(x) = \begin{cases} 2 + x^3, & x \leq 1 \\ 3x, & x > 1 \end{cases} \tag{1}$$

Determine whether $f$ satisfies the conditions of the mean-value theorem on the interval $[-1, 2]$ and, if so, find the values of $c$ guaranteed by the theorem.