

# Calculus I Sample Final Exam

1

1. Compute

$$\int 4x^3 + \frac{1}{x^2} dx.$$

2. Compute  $\int_0^\pi \cos^3 x \sin x dx$ .

3. Compute

$$a) \lim_{x \rightarrow 0} \frac{x^3 + 7x^2 - x}{5x^3 + 10x^2 + x} \qquad b) \lim_{x \rightarrow \infty} \frac{x^3 + 7x^2 - x}{5x^3 + 10x^2 + x}$$

4. Classify (removable, jump, infinite) the discontinuities of the piecewise defined function

$$f(x) = \begin{cases} \frac{1}{x} & \text{for } x > 0 \\ \sqrt{x^2 + 3} & \text{for } 0 > x > -1 \\ x + 3 & \text{for } -2 \leq x < -1 \\ \frac{1}{x} & \text{for } x < -2 \end{cases} \quad (1)$$

5. Compute the derivative of  $f(x) = \left(\frac{x+1}{x^2+1}\right)^3$  at  $x = 1$ .

6. A particle is moving around the ellipse  $3x^2 + 12y^2 = 51$ . As it passes through the point  $(1, -2)$  its  $y$ -coordinate is decreasing at the rate of 4 units per second. How fast is the  $x$ -coordinate changing at this time?

7. Let  $f(x) = x^2(1-x)$ . Draw the sign chart of  $f'(x)$ , find the local extreme VALUES.

8. Draw the graph of  $f(x) = x + \frac{2x}{x-1}$ . Indicate the increasing, decreasing, concave-up and concave-down intervals. Indicate the asymptotes.

9. Compute the area of the region bounded by  $x = y^2$  and  $x - y = 2$ .

10. Use disc method to compute the volume of a ball of radius  $r$ .